21.

$$f(x) = \frac{1}{x+1}\Big|_{x=0} = 1$$

$$f'(x) = -\frac{1}{(x+1)^2}\Big|_{x=0} = -1$$

$$f''(x) = \frac{2}{(x+1)^3}\Big|_{x=0} = 2$$

$$f'''(x) = \frac{-6}{(x+1)^4}\Big|_{x=0} = -6$$

$$f^{iv}(x) = \frac{24}{(x+1)^5}\Big|_{x=0} = 24$$

around $x=0$

$$P(x) = f(0) + f'(0)\,x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + a_4x^4 + \dots$$

$$P'(x) = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 \Big|_{x=0} = a_1$$

$$P''(x) = 2a_2 + 6a_3x + 12a_4x^2 \Big| x=0 = 2a_2$$

$$P'''(x) = 6a_3 + 24a_4\,x \Big| x=0 = 6a_3$$

$$f^{iv}(x) = 24a_4 \Big|_{x=0} = 24a_4$$

Feb 28-12:02 PM

$$\frac{1}{1-x}$$

$$
\begin{array}{r|l}
 & 1 + x + x^2 + \dots \\
\hline
1-x & 1 + 0x \\
 & \underline{1 - x} \\
 & \quad x \\
 & \quad \underline{x - x^2} \\
 & \qquad x^2 \\
 & \qquad \underline{x^2 - x^3} \\
 & \qquad\quad x^3
\end{array}
$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3$$

Example 2

$$\sum_{n=0}^{\infty} n!\,x^n$$

$$\lim_{n\to\infty}\left|\frac{(n+1)!\,x^{n+1}}{n!\,x^n}\right| < 1$$

$$\lim_{n\to\infty}\left|(n+1)x\right| < 1$$

No radius of convergence

$$x\lim_{n\to\infty}|n+1| < 1$$

$$\infty$$

Feb 28-12:22 PM

Ex 5 $\sum_{n=1}^{\infty} \frac{x^n}{n}$

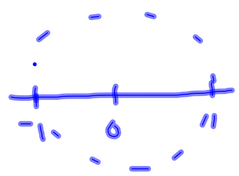
$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{n+1} \cdot \frac{n}{x^n} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{n \cdot x}{n+1} \right| < 1$$

$$|x| \lim_{n\to\infty} \left| \frac{n}{n+1} \right| < 1$$

$$|x| < 1$$ Radius of Convergence

$-1 < x < 1 \rightarrow$ Interval of Convergence



Check your endpoints: plug 1 into x

$x=1$ $\sum \frac{1^n}{n} = \sum \frac{1}{n}$ Divergent p-series test

$x=-1$ $\sum \frac{(-1)^n}{n}$ Convergent alternating series test

Final Answer Convergent on

Radius of Convergence $= 1$ $-1 \le x < 1$

5-10 11-31 odd
41, 43, 45-48, 63, 64